Profit maximization and cost minimization

A firm operates with the following production function:

$$f(K, L) = \sqrt{min\{K, L\}}$$

1. Find the scale returns
2. Find the unconditioned factor demands
3. Find the level of production (if it exists) that maximizes the firm's profits.
4. Find the conditioned demands of the factors
5. Find the minimum cost function

Solution

1. We calculate the degree of homogeneity:

$$f(hK, hL) = \sqrt{min\{hK, hL\}} = \sqrt{hmin\{K, L\}} = \sqrt{h}\sqrt{min\{K, L\}} = h^{1/2} f(K, L)$$

   The function has decreasing scale returns since the degree of homogeneity is less than 1. This means that an increase in K and L generates a non-proportional (and smaller) increase in production.

2. I state the profit function:

$$B = PQ - C = P\sqrt{min\{K, L\}} - (Kr + wL)$$

   One of the conditions to be met is that $K = L$, then I can state:

$$B = P\sqrt{K} - K(r + w)$$

   With the first order condition:

$$B'_K = P\frac{1}{2}K^{-1/2} - r - w = 0$$

$$K^{-1/2}\frac{1}{2} = \frac{r + w}{P}$$

$$(\frac{P}{2(r + w)})^2 = K$$

   This condition also applies to L :

$$(\frac{P}{2(r + w)})^2 = L$$

3. The level of production that maximizes the firm's profits is obtained by replacing the unconditioned demands in the production function:

$$f = \sqrt{min\{K, L\}} = \sqrt{(\frac{P}{2(r + w)})^2} = (\frac{P}{2(r + w)})$$

4. For the unconditioned demands, what I do is minimize the costs, subject to a minimum production: $\bar{y}$, and considering that $K = L$

$$L = Kr + Kw + \lambda(\bar{y} - \sqrt{K})$$

$$L'_K = r + w - \lambda\frac{1}{2}K^{-1/2} = 0$$

$$L'_\lambda = \bar{y} - \sqrt{K} = 0$$

   From the second equation:

$$\bar{y} - \sqrt{K} = 0$$

$$(\bar{y})^2 = K$$

   Therefore, the unconditioned demands:

$$K = L = (\bar{y})^2$$

   And the minimum cost:

$$C = rK + wL = (\bar{y})^2(w + r)$$